Critical level spacing distribution of two-dimensional disordered systems with spin-orbit coupling

1995 J. Phys.: Condens. Matter 7 L377

(http://iopscience.iop.org/0953-8984/7/28/002)

J. Phys.: Condens. Matter 7 (1995) L377–L381. Printed in the UK

LETTER TO THE EDITOR

Critical level spacing distribution of two-dimensional disordered systems with spin–orbit coupling

L Schweitzer† and I Kh Zharekeshev‡§

† Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, Germany
‡ Institut für Theoretische Physik, Universität Hamburg, Juingiusstrasse 9, D-20355 Hamburg, Germany

Received 15 March 1995, in final form 9 May 1995

Abstract. The energy level statistics of 2D electrons with spin–orbit scattering are considered near the disorder-induced metal–insulator transition. Using the Ando model, the nearest-level-spacing distribution is calculated numerically at the critical point. It is shown that the critical spacing distribution is size independent and has a Poisson-like decay at large spacings as distinct from the Gaussian asymptotic form obtained by the random-matrix theory.

Recently, the statistical description of electronic properties at the critical region near the Anderson transition in disordered systems has been a subject of great interest. Universal behaviour was proposed [1, 2] for the variance of the ensemble-averaged number of energy levels in a given interval, $\langle(\delta N)^2\rangle = \langle N^2\rangle - \langle N\rangle^2$, and for the nearest-level-spacing distribution $P(s)$. A relation $\langle(\delta N)^2\rangle = a\langle N\rangle$ could be extracted from numerical calculations for the three-dimensional Anderson model, while for $P(s)$ a universal combination of the well known Wigner surmise (small spacings) and the Poisson distribution (large spacings) was proposed. Recent analytical calculations [3, 4] suggested, however, another universal relation $\langle(\delta N)^2\rangle = a_{d\beta}\langle N\rangle^{\nu}/\beta$ near the critical transition (mobility edge) and the corresponding $P(s)$ was different from both the Poisson and the Wigner limit. Universal means here that the result depends only on the symmetry class β, the correlation exponent $\nu = [d(1-\gamma)]^{-1}$ and the spatial dimension d. The statistics of energy levels are governed by the symmetry of the Hamiltonian belonging to certain universality classes of the corresponding Gaussian ensembles of random matrices: orthogonal ($\beta = 1$), unitary ($\beta = 2$) and symplectic ($\beta = 4$) [5].

In two-dimensional (2D) disordered systems a complete Anderson transition is expected only for the symplectic symmetry class. Two different models for describing the localization problem in two dimensions in the presence of spin–orbit coupling have been proposed [6, 7]. For the Ando model [8] the critical behaviour of the localization length was previously studied by means of the transfer-matrix method [9, 10, 11]. The statistics of energy levels for symplectic symmetry in connection with the results of the random-matrix theory (RMT) was considered in the Evangelou–Ziman model [12] and the distribution $P(s)$ at the transition point for this model has already been mentioned [13]. The multifractal properties of the critical eigenstates in the symplectic case were investigated in [14, 15].

In this paper we study the level statistics of electrons with spin–orbit coupling in 2D disordered systems. By using the Ando model we calculate numerically the correlations in

§ Permanent address: Kazakh State University, Almaty 480121, Kazakhstan, CIS.

0953-8984/95/280377+05$19.50

the exact one-electron energy spectrum at the metal–insulator transition. Our main result is that the probability density $P(s)$ of neighbouring levels in the critical region is universal, i.e. it does not depend on the size of the system, and has a novel form.

It is known that in the metallic phase $P(s)$ is very close to the Wigner surmise [16] appropriate for the symplectic symmetry class of the random Hamiltonians [17]

$$P_{\mathrm{GSE}}(s) = \frac{2^{18}}{3^6 \pi^3} s^4 \exp\left(-\frac{64}{9\pi} s^2\right) \tag{1}$$

where s is measured in units of the mean level spacing. In the strongly localized regime the energy spectrum is completely uncorrelated, and the distribution of the level spacings obeys the Poisson law

$$P_{\mathrm{P}}(s) = \exp(-s). \tag{2}$$

In addition to these two universal distributions one can expect that there is a third form of $P(s)$, which corresponds exactly to the metal–insulator transition. This new universal statistics had already been found for the 3D disordered systems without spin–orbit scattering ($\beta = 1$) and confirmed numerically [2, 18]. An analogous result was recently obtained in the unitary case ($\beta = 2$) [19], where time-reversal symmetry is broken by the magnetic field.

It is reasonable to suppose that a similar scale-invariant universality of $P(s)$ also holds at the critical point for 2D disordered electrons with spin–orbit coupling. Thus, we expect that in the thermodynamic limit there exist three possible limiting situations for $\beta = 4$, namely, the Wigner surmise equation (1) for the metallic regime, the Poisson law equation (2) for the insulating regime and the critical distribution $P(s)$ at the mobility edge. Therefore, if $L \to \infty$, the level statistics change discontinuously twice, going from the delocalized regime to the transition point and, then, from the transition point to the localized regime.

In order to calculate the critical level statistics we start with the Hamiltonian of the Ando model [8]

$$H = \sum_{n,\sigma} \epsilon_n c^{\dagger}_{n\sigma} c_{n\sigma} + \sum_{n,m,\sigma,\sigma'} V(n,\sigma;m,\sigma')\, c^{\dagger}_{n\sigma} c_{m\sigma'} \tag{3}$$

where $c^{\dagger}_{n\sigma}$ and $c_{n\sigma}$ are the creation and annihilation operators of an electron at a lattice site $n = (x, y)$ with the spin component σ, and m denotes the sites adjacent to the site n. The on-site energy ϵ_n is randomly distributed around zero according to a box distribution with a width W. The parameter W specifies the degree of the disorder. The transfer matrices $V(n,\sigma;m,\sigma') = V_x, V_y$ depend on the direction

$$V_{x,y;x+1,y} = \begin{pmatrix} V_1 & V_2 \\ -V_2 & V_1 \end{pmatrix} \qquad V_{x,y;x,y+1} = \begin{pmatrix} V_1 & -\mathrm{i}V_2 \\ -\mathrm{i}V_2 & V_1 \end{pmatrix} \tag{4}$$

and describe the hopping between the nearest-neighbour sites in the lattice. The strength of the spin–orbit coupling is given by the parameter $S = V_2/V$, where $V = (V_1^2 + V_2^2)^{1/2}$ is taken as the unit of energy. In what follows we consider $S = 1/2$. It was earlier found by the transfer-matrix method [10] that the metal–insulator transition in the middle of the band $\varepsilon/V = 0$ occurs at a disorder $W_c/V = 5.74$.

Applying periodic boundary conditions, the exact discrete eigenvalue spectrum has been obtained from a numerical diagonalization of the Hamiltonian equation (3) using a Lanczos algorithm. For square lattices of linear size $L/a = 50$ (100) energy intervals $[-1, 0]$ ($[-0.5, 0]$) were chosen. Within these intervals the density of states is almost constant, $\rho = (\Delta L^2)^{-1} \approx 0.127$. Here, a is the lattice constant and Δ the mean level spacing. The eigenvalues taken from these intervals belong to the critical region where the correlation

length ξ is larger than the system size L. The total numbers of eigenvalues N were 94 672 (300 realizations) and 101 744 (160 realizations) for $L = 50a$ and $L = 100a$, respectively.

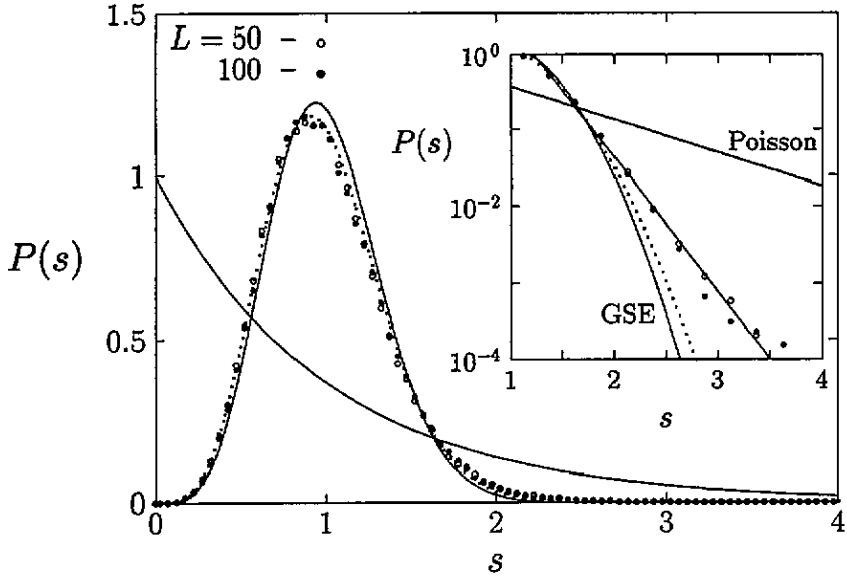

Figure 1. The level spacing distribution $P(s)$ for two different system sizes at the critical disorder $W_c/V = 5.74$. Solid lines correspond to $P_{GSE}(s)$ and $P_P(s)$, respectively. The dotted line is a fit of equation (5). The inset shows the large-s behaviour of $P(s)$. The straight line is a fit according to equation (6).

Figure 1 displays the level spacing distribution function $P(s)$ calculated for the different sizes of the system at the metal–insulator transition. One can see that independently of the system size the data lie on a common curve. This critical $P(s)$ is very close to P_{GSE} and passes the point $s_0 \approx 1.63$, where the two limiting distributions $P_{GSE}(s)$ and $P_P(s)$ cross. For small spacings $P(s) \propto s^4$. It is interesting to notice that our results differ from those obtained previously for smaller systems with the Evangelou–Ziman spin–orbit coupling model [13].

We applied the fitting function which was recently proposed for the description of the critical level distribution [3]

$$P(s) = Bs^4 \exp(-As^{2-\gamma}) \tag{5}$$

and found $A = 2.77 \pm 0.05$, $B = 17.8 \pm 0.8$ and $\gamma = 0.28 \pm 0.03$. Using a confidence level of 95% ($\alpha = 0.05$) the fitting procedure within the range $0 < s < 3$ where the statistics of our numerical data is rather good yielded $\chi^2 = 29.2$ which is less than the expected value $\chi^2_\alpha = 63.4$. Hence, the analytical formula (equation (5)) can be accepted within an approximate relative error of $(\chi^2/N)^{1/2} \approx 2\%$. Although in the range $0 < s < 3$ the calculated $P(s)$ is in good agreement with equation (5), the exponent γ obtained gives a different value of the correlation length exponent $\nu = [(1-\gamma)d]^{-1} \approx 0.7$ as compared to the value $\nu = 2.75$ obtained numerically using the transfer-matrix method [9, 10].

Moreover, for large spacings our results deviate markedly from the above equation (5) (inset of figure 1). Instead, the behaviour of $P(s)$ in the range of spacings $1.5 < s \lesssim 4$ is well described by the Poisson-like asymptotic form

$$P(s) \propto \exp(-\kappa s) \tag{6}$$

with a coefficient $\kappa = 4.0 \pm 0.2$. This decay is much slower than both the Gaussian decay of equation (1) and the intermediate decay from equation (5), but faster than that for the insulating regime, equation (2). A similar exponential tail of the critical $P(s)$ was also found in the 3D case without spin–orbit interactions [20] for which the coefficient κ is approximately a factor of two less. Such an asymptotic behaviour of the critical $P(s)$ is in good agreement with a suggestion about the Gaussian form for the distribution of the number of levels lying within a given energy interval [1, 21].

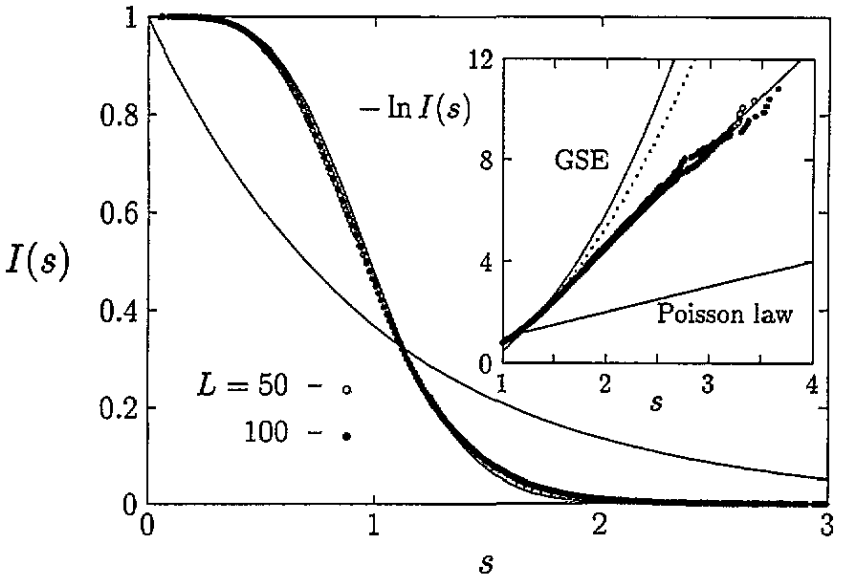

Figure 2. The integrated probability $I(s)$ at the critical disorder $W_c = 5.74$. The solid curves are $I_{GSE}(s)$ and $I_P(s)$ for the metallic and insulating phases, respectively. The dotted line is obtained from equation (5). The inset shows the large-s part of $I(s)$. The straight line fitting the data is $\ln I(s) = -4.0s + 3.6$.

In order to diminish the magnitude of relative fluctuations due to the limited number of realizations and to analyse the asymptotic behaviour of the level spacing distribution in detail, it is more convenient to consider a total probability function $I(s) = \int_s^\infty P(s')\,ds'$. This quantity implies a portion of spacings which are larger than a given s. It is clear that $I(0) = 1$ and $\int_0^\infty I(s')\,ds' = 1$ regardless of the disorder. In the strongly localized regime $I_P(s) = \exp(-s)$, and $I_{GSE}(s)$ can be calculated from the RMT. The results of the numerical calculations for the critical $I(s)$ are shown in figure 2. One again observes a discrepancy from both the GSE asymptotic and the $I(s)$ obtained from equation (5), particularly when s is large. The form of the critical $I(s)$ is not sensitive to the change of the lattice size, in analogy with $P(s)$. The Poisson tail as described by equation (6) is recovered for large spacings: $\ln I(s) \propto -s$. We have checked that the slope of the linear behaviour of $\ln I(s)$ at the transition does not depend on the width of the energy interval from which the levels are taken, as long as they belong to the critical region. In the range of very small probability ($s > 3$) the larger fluctuations observed are due to the insufficiency of the statistical data. However, the accuracy of the total calculated $I(s)$ in higher than that of $P(s)$. One should notice that the behaviour of the level spacing distribution obtained obviously does not reflect the information on the critical exponent ν and the dimensionality d in the form expressed by equation (5).

In conclusion, we have presented results of computer simulations of the nearest-level-spacing distribution $P(s)$ for 2D disordered systems in the presence of spin–orbit interaction. Exactly at the metal–insulator transition $P(s)$ and, consequently, the total probability of neighbouring spacings $I(s)$ do not depend on the system size and are different from the universal limiting distributions corresponding to the metallic and the insulating regime. They appear to exhibit critical behaviour at the disorder $W_c/V = 5.74$ and finite-size scaling properties around the critical point. The large-s parts of $P(s)$ and $I(s)$ obtained are shown to have a Poisson-like decay, so $\ln P(s) = -\kappa s$ where $\kappa \approx 4.0$ is larger than it is for the insulating regime ($\kappa = 1$).

We are grateful to B Kramer and B Shklovskii for helpful discussions. IKhZh thanks the DAAD for financial support during his stay at the University of Hamburg.

References

[1] Altshuler B L, Zharekeshev I Kh, Kotochigova S A and Shklovskii B I 1988 *Zh. Eksp. Teor. Fiz.* **94** 343 (Engl. Transl. 1988 *Sov. Phys.–JETP* **67** 625)
[2] Shklovskii B I, Shapiro B, Sears B R, Lambrianides P and Shore H B 1993 *Phys. Rev.* B **47** 11 487
[3] Aronov A G, Kravtsov V E and Lerner I V 1994 *Pis. Zh. Eksp. Teor. Fiz.* **59** 40 (Engl. Transl. 1994 *JETP Lett.* **59** 39)
[4] Kravtsov V E, Lerner I V, Altshuler B L and Aronov A G 1994 *Phys. Rev. Lett.* **72** 888
[5] Mehta M L 1991 *Random Matrices* (Boston, MA: Academic)
[6] Evangelou S N and Ziman T 1987 *J. Phys. C: Solid State Phys.* **20** L235
[7] Ando T 1988 *Surf. Sci.* **196** 120
[8] Ando T 1989 *Phys. Rev.* B **40** 5325
[9] Fastenrath U, Adams G, Bundschuh R, Hermes T, Raab B, Schlosser I, Wehner T and Wichmann T 1991 *Physica* A **172** 302
[10] Fastenrath U 1992 *Physica* A **189** 27
[11] Markoš P 1994 *J. Physique* **4** 551
[12] Evangelou S N 1989 *Phys. Rev.* B **39** 12 895
[13] Evangelou S N 1990 *Physica* A **167** 199
[14] Chalker J T, Daniell G T, Evangelou S N and Nahm I H 1993 *J. Phys. C: Solid State Phys.* **5** 485
[15] Schweitzer L 1995 *J. Phys.: Condens. Matter* **7** L281
[16] Wigner E P 1955 *Ann. Math.* **62** 548; 1957 *Ann. Math.* **65** 203
[17] Efetov K B 1983 *Adv. Phys.* **32** 53
[18] Hofstetter E and Schreiber M 1993 *Phys. Rev.* B **48** 16 979; 1994 *Phys. Rev.* B **49** 14 726
[19] Hofstetter E and Schreiber M 1994 *Phys. Rev. Lett.* **73** 3127
[20] Zharekeshev I Kh and Kramer B 1995 *Japan. J. Appl. Phys.* at press
[21] Fogler M M and Shklovskii B I 1995 *Phys. Rev. Lett.* at press